Reply to “Comments on `Diathermal heat transport in a global ocean model'”

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CORRESPONDENCE

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ABSTRACT

Hochet and Tailleux (2019), in a comment on Holmes et al. (2019), argue that under the incompressible Boussinesq approximation the "sum of the volume fluxes through any kind of control volume must integrate to zero at all times." They hence argue that the expression in Holmes et al. (2019) for the change in the volume of seawater warmer than a given temperature is inaccurate. Here we clarify what is meant by the term "volume flux" as used in Holmes et al. (2019) and also more generally in the water-mass transformation literature. Specifically, a volume flux across a surface can occur either due to fluid moving through a fixed surface, or due to the surface moving through the fluid. Interpreted in this way, we show using several examples that the statement from Hochet and Tailleux (2019) quoted above does not apply to the control volume considered in Holmes et al. (2019). Hochet and Tailleux (2019) then derive a series of expressions for the water-mass transformation or volume flux across an isotherm $\mathcal{G}$ in the general, compressible case. In the incompressible Boussinesq limit these expressions reduce to a form (similar to that provided in Holmes et al. 2019) that involves the temperature derivative of the diabatic heat fluxes. Due to this derivative, $\mathcal{G}$ can be difficult to robustly estimate from ocean model output. This emphasizes one of the advantages of the approach of Holmes et al. (2019), namely, $\mathcal{G}$ does not appear in the internal heat content budget and is not needed to describe the flow of internal heat content into and around the ocean.

We thank Hochet and Tailleux (2019) for their comment and interest in our paper. We believe that confusion has arisen here through a difference in interpretation of the term "volume flux" in the context of a control volume that is neither fixed in space nor follows the fluid motion. We start with a kinematic definition of the total flux of volume across a surface defined by a constant value of some property Θ that will later be taken as potential temperature (e.g., Groeskamp et al. 2019),

$$\mathcal{G} = \int_{\partial\mathcal{V}} (\mathbf{v} - \mathbf{v}_b) \cdot \hat{\mathbf{n}}\, dS. \tag{1}$$

In Eq. (1) $\partial\mathcal{V}$ denotes the surface defined by a constant value of Θ, $(\mathbf{v} - \mathbf{v}_b) \cdot \hat{\mathbf{n}}$ is the dia-surface velocity, being the difference between the motion of the fluid ($\mathbf{v}$) and the motion of the surface itself ($\mathbf{v}_b$), $\hat{\mathbf{n}}$ is a normal to the surface, and dS is an area-element on the surface. We take $\mathcal{G}$ as our definition of "volume flux across the

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Θ surface.'' With this definition, it is clear that the volume flux depends not only on the motion of the fluid but also on the motion of the surface. Critically, a nonzero volume flux across the surface can occur even in the absence of any fluid motion if the surface itself moves through the fluid (i.e., $\mathbf{v}=0$ but $\mathbf{v}_b \neq 0$). We believe this is the main point of difference between our perspective and that of Hochet and Tailleux (2019)—they interpret a volume flux as only occurring because of fluid motion [as clear from the discussion above their Eq. (4)], whereas we have not made that assumption.

To illustrate this point further, we consider the volume budget of a general, potentially time-dependent, volume $\mathcal{V}(t)$. The volume $\mathcal{V}(t)$ can be defined in terms of an integral of a volume element dV,

$$\mathcal{V} = \int_{\mathcal{V}} dV. \tag{2}$$

Taking the time derivative,

$$\frac{\partial \mathcal{V}}{\partial t} = \frac{\partial}{\partial t}\left(\int_{\mathcal{V}} dV\right), \tag{3}$$

$$= \int_{\partial \mathcal{V}} \mathbf{v}_b \cdot \hat{\mathbf{n}}\, dS, \tag{4}$$

where in the second line we have used the Leibniz integral rule to relate the change in volume to the movement of its bounding surface $\partial\mathcal{V}(t)$ through the motion of the points on that bounding surface $\mathbf{v}_b$ and the normal to that bounding surface $\hat{\mathbf{n}}$. Now the problem with Eq. (4) of Hochet and Tailleux (2019) when applied to a general volume is clear; $\partial\mathcal{V}/\partial t$ is determined by the movement of the volume's bounding surface $\mathbf{v}_b$, which is in general not equal to the fluid velocity $\mathbf{v}$. Hochet and Tailleux (2019) may have had in mind the case of a control volume whose boundaries are fixed in three-dimensional space. In this case $\mathbf{v}_b = 0$ and $\partial\mathcal{V}/\partial t = 0$ by construction (even if in the compressible case $\int_{\partial\mathcal{V}} \mathbf{v}\cdot\hat{\mathbf{n}}\, dS \neq 0$). Another example to consider is that of a material volume, say $\mathcal{V}_{\mathrm{mat}}$, bounded by a material surface that follows the motion of the fluid such that $\mathbf{v}_b = \mathbf{v}$. In this case,

$$\frac{\partial \mathcal{V}_{\mathrm{mat}}}{\partial t} = \int_{\partial \mathcal{V}_{\mathrm{mat}}} \mathbf{v}_b \cdot \hat{\mathbf{n}}\, dS, \tag{5}$$

$$= \int_{\partial \mathcal{V}_{\mathrm{mat}}} \mathbf{v} \cdot \hat{\mathbf{n}}\, dS, \tag{6}$$

$$= \int_{\mathcal{V}_{\mathrm{mat}}} \nabla \cdot \mathbf{v}\, dV, \tag{7}$$

where in the last line we have used Gauss's theorem. If the fluid were incompressible then $\nabla\cdot\mathbf{v}$ vanishes, meaning that in an incompressible Boussinesq fluid any material volume is conserved.

We now return to the specific volume considered in Holmes et al. (2019),

$$\mathcal{V}(\Theta, t) = \iiint_{\Theta'(x,y,z,t)>\Theta} dV, \tag{8}$$

defined as the volume of the ocean warmer than some temperature Θ bounded by the Θ isotherm and the ocean surface. This volume is neither fixed in space, nor is it a material volume. Volume fluxes can occur across the surfaces bounding this volume, the Θ isotherm and the ocean surface, that are not linked solely to the motion of the fluid at those surfaces. This is demonstrated by two examples in Fig. 1. Figure 1a examines the case where a motionless ($\mathbf{v}=0$ everywhere) incompressible ocean is uniformly heated through penetrating shortwave radiation, resulting in a deepening of the Θ isotherm ($\mathbf{v}_b < 0$), a nonzero volume flux across that isotherm $\mathcal{G}(\Theta, t) > 0$, and an increase in the volume $\mathcal{V}(\Theta, t)$. The nonzero dia-surface velocity (or in this case *diathermal* velocity) associated with the nonzero $\mathcal{G}$ can only occur in the presence of diabatic processes (in Fig. 1a, warming via penetrating shortwave radiation), which result in a change in the temperature of constituent fluid particles. In Holmes et al. (2019) these diabatic processes are provided by the convergence of heat fluxes associated with vertical mixing, numerical mixing, or surface forcing, as expressed by Eq. (14) of Holmes et al. (2019) [or Eq. (32) of Hochet and Tailleux 2019]. In contrast, there is no volume flux across the surface defined by the initial position of the Θ isotherm (black dashed line in Fig. 1a), as this surface is fixed in time and the fluid is motionless.

It is important to note that the ocean surface is also neither fixed in space or time, nor a material surface in the presence of nonzero surface mass fluxes. Nonbreaking surface gravity waves drive undulations in the free surface, which do not drive any volume fluxes across the sea surface and therefore do not change the volume $\mathcal{V}$ (i.e., $\mathbf{v}_b\cdot\hat{\mathbf{n}} = \mathbf{v}\cdot\hat{\mathbf{n}}$ in this case). However, in the presence of precipitation fluid does move across the surface, resulting in an increase in sea surface height and in the volume $\mathcal{V}$ (provided that the precipitated water is warmer than Θ, Fig. 1b).

In conclusion, providing that the phrase ''volume flux across a surface'' is defined by Eq. (1), meaning that a nonzero volume flux can occur either because of fluid motion across a fixed surface or the movement of the surface through the fluid itself, then we do not believe

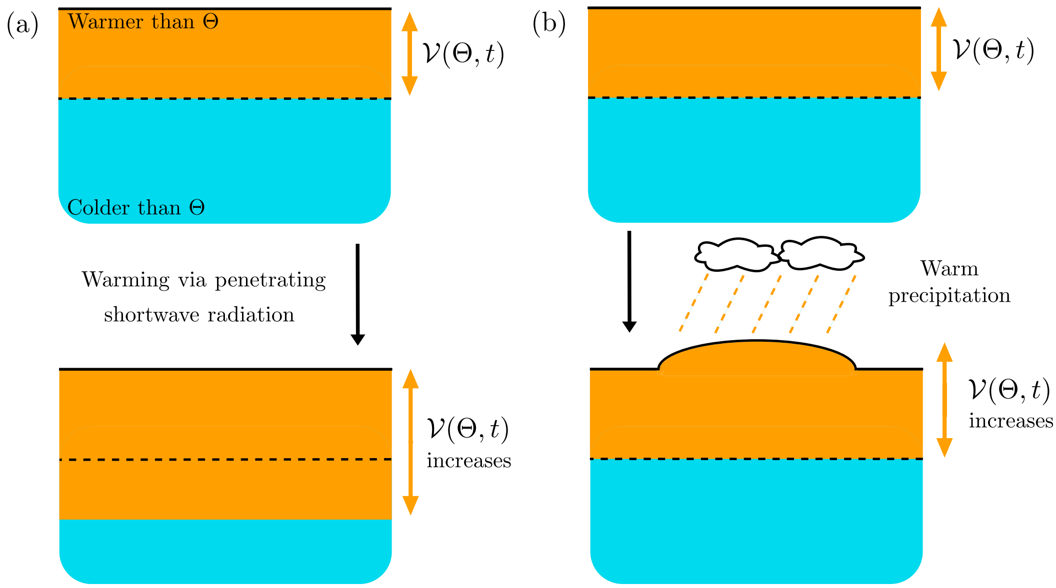

FIG. 1. Two examples demonstrating that the bounding surface $\partial\mathcal{V}$ of the volume $\mathcal{V}(\Theta, t)$ defining all fluid warmer than a given temperature Θ and within the ocean (orange volume) is neither fixed in space nor a material surface. Both cases consider an incompressible motionless ocean ($\mathbf{v} = 0$ everywhere). (a) The ocean is warmed uniformly due to penetrating shortwave radiation. The Θ isotherm moves downward as the fluid warms, meaning that there is a volume flux across the Θ isotherm $\mathcal{G}(\Theta, t) > 0$ and $\mathcal{V}(\Theta, t)$ increases. However, there is no volume flux across the fixed surface defined by the initial position of the Θ isotherm (dashed line). (b) Precipitation, where the precipitated water is warmer than Θ, moves across the ocean surface and adds to the volume $\mathcal{V}$.

there is any confusion or ambiguity in Holmes et al. (2019) (or indeed Walin 1982). The volume considered in Holmes et al. (2019) is neither fixed in space nor a material surface, and thus its time variability cannot be linked solely to the motion of the fluid itself. For this reason, we do not believe it is necessary to regard the volume $\mathcal{V}$ as the Boussinesq mass. Of course, if the incompressible Boussinesq assumption is not made, then (as stated in the second footnote on p. 144 of Holmes et al. 2019) it is more natural to consider the mass, rather than volume, enclosed by the isotherm and the surface. With compressible dynamics, as discussed by Hochet and Tailleux (2019), changes in the mean density of the region can result in changes in its volume without any fluxes of mass through the bounding surface. However, whether the incompressible Boussinesq approximation is made or not, we emphasize that one of the advantages of the approach used in Holmes et al. (2019) is that one does not have to consider the water-mass transformation term $\mathcal{G}$ to understand the flow of internal heat content within the ocean. The transformation $\mathcal{G}$ does not appear in the internal heat content budget considered by Holmes et al. (2019), which instead depends directly on the surface heat flux and diffusive fluxes of heat across isotherms. This is an advantage as $\mathcal{G}$ is a differentiated quantity that can be more difficult to robustly estimate from model simulations.

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